

## ZÁPORNÉ ČÍSLA A OPAČNÉ ČÍSLA

Začneme nasledujúcim citátom z Wikipédie:

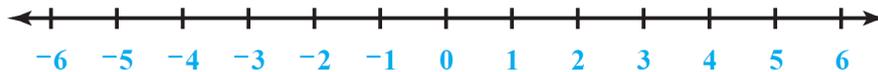
“Some elementary teachers use raised plus and minus signs before numbers to show they are positive or negative numbers. For example subtracting  $-5$  from  $3$  might be read as positive three take away negative  $5$  and be shown as  $3 - -5$  becomes  $3 + 5 = 8$ , or even as  $+3 - -5$  becomes  $+3 + +5$  which is  $+8$ .”

[http://en.wikipedia.org/wiki/Plus\\_and\\_minus\\_signs](http://en.wikipedia.org/wiki/Plus_and_minus_signs)

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S takýmto spôsobom zápisu záporných čísel sa môžeme stretnúť v niektorých zahraničných učebniciach. Na ilustráciu uvidíme tri ukážky.

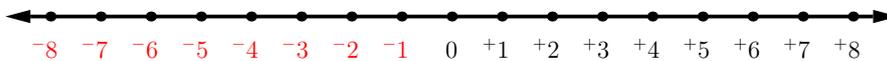
Učebnica z roku 2004:



Impact Mathematics: Algebra and More, Course 1, Student Edition,  
Glencoe/McGraw-Hill, 2004. ISBN 0-07-860909-7

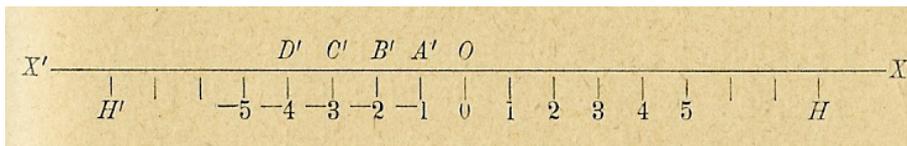
[http://www2.lhric.org/poCantico/math/Course\\_1/chap02-e.pdf](http://www2.lhric.org/poCantico/math/Course_1/chap02-e.pdf)

Učebnica z roku 1985:



Robert E. Eicholz & all, Addison-Wesley Mathematics, 1985. ISBN 0-201-24800-X

Učebnica z roku 1900:



Luis Octavio de Toledo (1900), Elementos de aritmética universal. Calculatoria.  
Madrid: Imprenta Fortanet.

[http://repositorio.ual.es/jspui/bitstream/10835/743/1/Art\\_17\\_298\\_eng.pdf](http://repositorio.ual.es/jspui/bitstream/10835/743/1/Art_17_298_eng.pdf)

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V skutočnosti symbol mínus má tri významy. Napr. v knihe [5] sa píše:

The *minus* sign can mean three different things, depending on the context.

- It can mean **negative**. In front of a positive number, and only there, it means negative. Example:  $-2$  can mean negative 2.
- It can mean **opposite**. The opposite of a number is what you add to it to get zero. Example:  $-2$  can mean the opposite of 2, which is negative 2, since  $2 + -2 = 0$ . Likewise,  $-x$  means the opposite of  $x$ , and  $x + -x = 0$ .
- It can mean **subtract**. Between two expressions, it means subtract the second expression from the first one. For example,  $x - 3$  means subtract 3 from  $x$ .

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V učebnici [2] na strane 178 sa píše:

Each integer has an **opposite**.

**+6** is the opposite of **-6**.

**-4** is the opposite of **+4**.

The opposite of **0** is **0**.

A na strane 182 sa píše:

Positive integers are the same as whole numbers greater than zero. Positive rational numbers are the same as fractional numbers greater than zero. We can omit the raised plus sign (+) before positive integers and positive rational numbers.

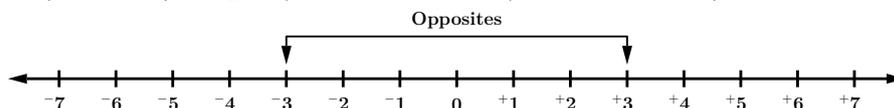
$$+7 = 7$$

$$+\frac{1}{4} = \frac{1}{4}$$

$$+6.4 = 6.4$$

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Dvojice navzájom opačných čísel môžeme vyznačiť na číselnej osi:



[http://web.cerritos.edu/dford/SitePages/Math40\\_S14/Math40S14Lecture3-Integers.pdf](http://web.cerritos.edu/dford/SitePages/Math40_S14/Math40S14Lecture3-Integers.pdf)

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“Working with Signed Numbers” je názov štvrtej kapitoly v učebnici:

Impact Mathematics: Algebra and More, Course 2, Student Edition,  
Glencoe/McGraw-Hill, 2004. ISBN 0-07-860920-8

[http://www2.lhric.org/poCantico/math/Course\\_1/chap04-e.pdf](http://www2.lhric.org/poCantico/math/Course_1/chap04-e.pdf)

Záporné čísla a opačné čísla sú tam vysvetlené formou tohto fiktívneho dialógu:

Zoe and Darnell have different ways of thinking about negative numbers.

Zoe: When I see a negative number, I think about where it is on the number line.

Negative numbers are to the left of 0.

For  $-4$ , the negative sign tells me to move to the left of 0, and the 4 tells me to move 4 units.

So  $-4$  is 4 units to the left of 0

Darnell: I think of negative numbers as opposites of positive numbers. When I see  $-2$ , I think, “the opposite of 2.”

Thinking about opposites helps me understand expressions with more than one negative sign. For example, I think of  $-(-3)$  as the opposite of  $-3$ .

And since the opposite of  $-3$  is 3, I know that  $-(-3)$  is equal to 3.

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Násobenie ako opakované sčítanie:

$$(-1) \cdot 1 = -1$$

$$(-1) \cdot 2 = (-1) + (-1) = -2$$

$$(-1) \cdot 3 = (-1) + (-1) + (-1) = -3$$

$$(-1) \cdot 4 = (-1) + (-1) + (-1) + (-1) = -4$$

Na základe toho môžeme vysloviť hypotézu, že pre každé prirodzené číslo  $n$  platí

$$(-1) \cdot n = -n.$$

Môžeme to overiť matematickou indukciou. Návod:

$$(-1) \cdot (k + 1) = (-1) \cdot k + (-1) \cdot 1 = (-k) + (-1) = -(k + 1).$$

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Materiál [1], ktorý je dostupný na adrese:

<http://homepages.math.uic.edu/~jbaldwin/pub/assoc.ps>

uvádza vzťah medzi binárnym mínus a unárnym mínus v tvare

$$a - b = a + -b$$

a tiež ukazuje, že platí

$$-a = (-1) \cdot a.$$

Naozaj, z distributívneho zákona máme

$$a + (-1) \cdot a = 1 \cdot a + (-1) \cdot a = (1 + -1) \cdot a = 0 \cdot a = 0.$$

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Pozrime sa na odlišné označovanie opačných čísel vo francúzskej literatúre.

### Opposé d'un nombre relatif

L'opposé d'un nombre relatif est le nombre relatif de signe contraire et de même distance à 0. L'opposé d'un nombre  $x$  se note:  $\text{opp}(x)$ .

L'opposé d'un nombre relatif est le nombre relatif obtenu en changeant son signe.

L'opposé de  $x$  se note:  $\text{opp}(x)$ .

Exemple:  $\text{opp}(+5) = -5$ ,  $\text{opp}(-7) = +7$ .

Soustraire un nombre revient à ajouter son opposé:

$$a - b = a + \text{opp}(b).$$

<http://michaudbonnet.ovh.org/archives/livre5.pdf>

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Pre ďalšie štúdium odporúčame tiež nasledujúci materiál:

James Tanton: WHY IS NEGATIVE TIMES NEGATIVE POSITIVE?

[http://www.jamestanton.com/wp-content/uploads/2012/03/Curriculum-Newsletter\\_October-2012.pdf](http://www.jamestanton.com/wp-content/uploads/2012/03/Curriculum-Newsletter_October-2012.pdf)

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### LITERATÚRA

- [1] John Baldwin, *What is a minus sign anyway?* (2001), preprint.
- [2] Robert E. Eicholz & all, *Addison-Wesley Mathematics*, Addison-Wesley, 1985, ISBN 0-201-24800-X.
- [3] Lenka Kapustová, *Epistemologické prekážky žiakov spojené s prijímaním záporných čísel*, Dizertačná práca, FMFI UK Bratislava, 2012.
- [4] Alexander Maz-Machado, Luis Rico-Romero, *Negative numbers in the 18th and 19th centuries: phenomenology and representations*, Electronic Journal of Research in Educational Psychology No. 17, **7(1)** (2009), 537–554, ISSN 1696-2095.
- [5] Anita Wah, Henri Picciotto, *Algebra: Themes, Concepts, Tools*, Creative Publications, 1994, ISBN 1-56107-251-6.