DEVELOPMENT OF INQUIRY SKILLS TO ANALYSE AND DETERMINE RELATIONSHIPS BETWEEN VARIABLES IN MATHEMATICS TEACHING USING GEOGEBRA

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Abstract

The official documents of the European Commission focused on mathematics education emphasize that inquiry-based learning is an important innovative strategy of mathematics teaching. The implementation of inquiry-based learning is in the focus of mathematics and science educators in Slovakia. Appropriate application of inquiry-based learning in mathematics teaching should stimulate the development of students' inquiry skills. The skills to analyse and determine relationships between variables are among the basic inquiry skills. Development of these inquiry skills requires that students initially work with simple models that would lead them to explore and use numerical and graphical representations of dependencies between variables. Active engagement of students in appropriately prepared inquiry activities should encourage students' understanding of connections between numerical, graphical and symbolic representations of functional dependencies.

Our research is currently focused on implementation of inquiry approaches to mathematics teaching and exploring the impact of inquiry-based learning on developing students' inquiry skills. Within the framework of the national project, we prepared inquiry activities for various mathematical topics and teachers have tried the inquiry activities in mathematics teaching. A pre-test to diagnose the level of development of selected inquiry skills and understanding the concept of function was given to students from six grammar schools in Slovakia at the beginning of the project. The sample consisted of 216 students. The post-test was given to the same students after one year, during which the teachers should use several inquiry activities for teaching functions and other topics. The paper analyses and interprets the results of solving three tasks focused on creating tables and graphs and characterizing the relationship between variables.

Two inquiry activities are presented in the second part of the paper. Inquiry activity focused on temperature transfer was designed for teaching linear function. GeoGebra is used to create a stimulating interactive learning environment in which feedback on student performance control is also implemented. An important part of the activity is a model for identifying matching temperature pairs in the Celsius and Fahrenheit scales, by which students can create a table and analyse the relationship between the obtained data. The second inquiry activity focused on investigating the relationship between side lengths of rectangles with the same area was designed based on the identified lacks in students' inquiry skills. By sketching several rectangles with the given area and creating a table with their side lengths, students can explore the dependence between the side lengths of the rectangles in a dynamic graphical model created in GeoGebra. The dynamic model allows students not only to explore the functional dependency between variables, but also to find rectangles having the minimum circumference for the given area.

Keywords: Pedagogical Innovations in Education, Problem and Project-Based Learning, Emerging Technologies in Education, Technology-Enhanced Learning.

1 INTRODUCTION

In the last two decades, there is a significant effort to implement inquiry-based and problem-based approaches into mathematics and science education. These disciplines together with engineering and technology form the basic components of STEM education. STEM education has a significant impact on the prosperity of society. Improving STEM education requires a deep consideration of mathematics.

Mathematics teachers could collaborate with other STEM subjects to integrate inquiry approaches into education [1]. The official documents of the European Commission [2] focused on mathematics education emphasize that inquiry-based learning (IBL) is an important innovative strategy of mathematics teaching.

IBL is effective constructivist approach to learning. This approach is derived from inquiry methods characteristic for the work of scientists. Inquiry in teaching, like scientific research, is aimed at finding answers to research questions, explaining the obtained results and using appropriate arguments to justify the findings. IBL enables students to design and conduct their own scientific investigations [3]. The teacher's role in IBL is one of organizing and facilitating learning, rather than transmitting knowledge [4]. IBL is currently widely recommended for mathematics and science teaching as a way to increase motivation and student's achievement [2]. The teacher should pose a motivating inquiry question at the beginning of the inquiry to arouse students' interest. The teacher is expected to engage the students in rich mathematical activities based on challenging mathematical tasks [5]. In a similar way, Rassmussen and Kwon [6] develop these possibilities, seeing the great advantage of IBL in enabling teacher the better positions to build on students' thinking by posing new questions and tasks. Students should already have initial experience and knowledge of some aspects of the explored phenomena so that they can build on them and use them to discover, understand and explain new findings.

2 DEVELOPMENT OF STUDENTS' INQUIRY SKILLS AT THE GRAMMAR SCHOOLS

Experiment or modelling should be an important part of inquiry activities. IBL fosters observations followed by experimentation, modelling, and justification of findings [7]. The student should learn to suggest a procedure and carry out an experiment to test a hypothesis or use modelling to obtain data, look for ways to process and analyse them, determine and explain the causal relationships in the observed phenomena.

Tendencies to innovate and improve science and mathematics education are reflected in a number of national and international projects. Recently, we participate in solving Slovak national project "IT Academy – education for 21st century". It supports the development of IT industry through the changes in educational system at primary schools, secondary schools, and universities. Proposed innovations enhance the quality of education in IT, mathematics, science, and related technology areas that have as the main focus ICT. Further, they improve the motivation of pupils and students to study informatics and ICT, and help to develop scientific abilities of both, pupils, and students.

From current modern ICT, the program GeoGebra offers great opportunities for modelling and exploring mathematical relationships. This program belongs among the most used programs in mathematics teaching. GeoGebra integrates algebraic and geometric tools and it provides great possibilities to visualize and investigate mathematical structures and relationships in mathematics teaching [8]. GeoGebra provides great possibilities to create stimulating interactive learning environment which enables students explore properties of objects and functional dependencies between quantities. GeoGebra can help students to explore, conjecture, visualize and explain mathematical relationships. It allows generating appropriate examples and counterexamples from which students can find out patterns, make conjectures, and develop arguments [9].

Appropriately prepared inquiry learning activities, that engage students and arouse their interest in active inquiry, could promote development of students' inquiry skills. In identifying the inquiry skills developed in the mathematics teaching, we used the classification of inquiry skills proposed by Van den Berg [10], in which the inquiry skills are divided into five categories derived from the basic stages of inquiry process. Active mathematics learning should lead to the development of these important inquiry skills:

- use experimentation/modelling to obtain information about the investigated objects and relationships,
- transform the obtained data into clearly designed tables, diagrams, graphs,
- analyse and determine the relationships between variables and express them using symbolic notations,
- find appropriate arguments to defend and justify the obtained results,
- draw conclusions, generalize the discovered findings.

In this paper, we focused on the inquiry skills to analyse and determine relationships between variables. Tasks for testing inquiry skills to analyse relationships between variables are also included in international PISA testing. For example, applying a linear relationship between variables and comparing function values these tasks required, [11]: *Jenn works at a store that rents DVDs and computer games. At this store the annual membership fee costs 10 zeds. The DVD rental fee for members is 2.50 zeds for one DVD and the fee for non-members is 3.20 zeds for one DVD.*

Question 1: *Troy was a member of the DVD rental store last year. Last year he spent 52.50 zeds in total, which included his membership fee. How much would Troy have spent if he had not been a member but had rented the same number of DVDs?*

Question 2: *What is the minimum number of DVDs a member needs to rent so as to cover the cost of the membership fee?*

Students can solve these questions using several strategies. The expected algebraic approach to the solution is based on the use of two functions. When x represents the number of borrowed DVDs, can be total fee for borrowed DVDs for members and for non-members interpreted using linear functions: f: $y = 2.5x + 10$ and g: $y = 3.2x$.

The function f enables students calculate the number of DVDs that Troy borrowed last year and the function g express the total fee for borrowed DVDs for non-members of the DVD rental store. The average percentage of successful students within the OECD countries in the question 1 was approximately 40 %.

When solving question 2, it is possible to compare functions f, g and determine the value of the variable x, for which the values of both functions are equal. The expected arithmetic approach to the solution is based on the finding that for a single DVD, a member saves 0.70 zeds. Because a member has already paid 10 zeds, he should borrow at least 10/0.7 DVDs (so 15 DVDs). The average percentage of successful students within the OECD countries in question 2 was approximately 17 %.

3 EVALUATION OF THE DEVELOPMENT LEVELS OF HIGH SCHOOL STUNDETS' INQUIRY SKILLS

We prepared a test to diagnose the development levels of high school students' the first four of mentioned above inquiry skills. Questions in the test were aimed to explore relationships in situations from various areas of school mathematics. The test contains ten questions. Seven questions were opened and three questions had the form of multiple choice question. Solving the opened questions required to write the steps of the student's solution. In the three multiple choice questions, students were also directly asked to justify their answer. We selected three questions from this test, which were aimed at testing inquiry skills to analyse and determine the relationships between variables.

Question 1 was focused on testing inquiry skills to use experimentation/modelling to obtain information about the investigated objects and relationships and transform the obtained data into clearly designed tables, diagrams, graphs: *Draw at least three different rectangles in a square grid so that each has an area of 12 cm^2. Write the lengths of the sides a, b of the sketched rectangles in the table. For each such rectangle can be applied the equation a · b = 12. Select a graph that expresses this dependence between sides a, b. Justify your answer.*

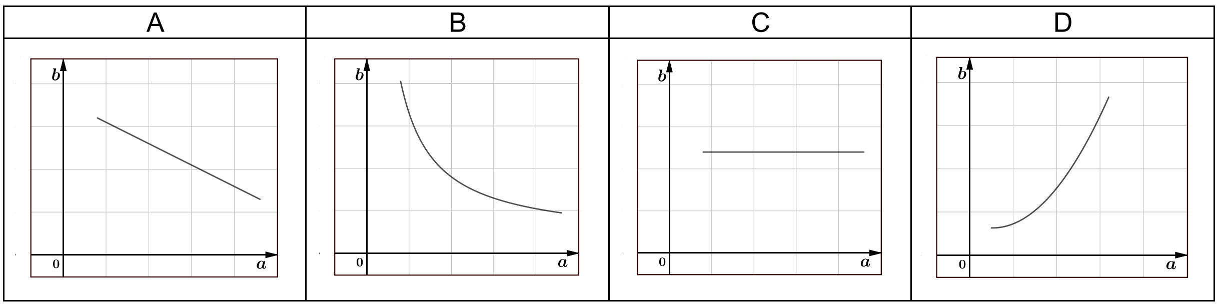

Figure 1. Graphs in the question 1.

Question 2 was focused on the use of graphs for determination of the relationships between variables and their representation using symbolic notations: *Draw a graph of a linear function that passes through the points P_x [c, 0] and P_y [0, 2] where* $c < 0$. Equation *$y = ax + b$ is the rule of the linear function. Which of the following statements about the number a is true?*

a) a > 0

b) a < 0

c) a = 0

d) the number a is not defined.

The third question was analogous to the question mentioned in the previous part of this paper from PISA testing. The question was focused on inquiry skills to analyse and determine the relationships between variables, express them using symbolic notations and compare the functions values: *Peter exercises in a fitness centre, in which one training costs 4,50 €. He paid yearly membership fee of 50 €. He received discounted fee of 3.30€ for each training to the end of the year. At least how many trainings should Peter realize by the end of the year to cover the cost of the membership fee?*

At the beginning of the project we selected six grammar schools in Slovakia which were participated in the verifying the prepared innovative teaching materials in mathematics, informatics and science teaching. A pre-test to diagnose the level of development of selected inquiry skills and understanding the concept of function was given to students in the selected classes of the first year of study. The same test was given to the same classes after one year, during which the teachers should use at least three inquiry learning activities for teaching linear and quadratic functions and other topics from the another subjects. After selecting the students who solved both tests, 216 students remained in the sample.

For our research we determined three hypotheses:

- H1: By applying inquiry approaches to teaching, there is a statistically significant difference in the overall increase in the level of development of students' selected inquiry skills.
- H2: By applying inquiry approaches to teaching, there is a statistically significant difference in the overall increase in the level of development of students' inquiry skills to analyse and determine relationships between variables.
- H3: By applying inquiry approaches to teaching, there is not a statistically significant difference in the overall increase in the level of development of students' inquiry skills to analyse and determine relationships between variables by gender.

By verifying the normality of the distribution using the Lilliefors test, we statistically verified the established hypotheses. To verify them, we used paired t-test, F-test two-sample for variances and t-test two-sample for means of normal distribution.

4 RESULTS

The hypothesis H1 was confirmed on the significance level of 0,05.

Table 1. t-Test: Paired Two Sample for Means.

	Pre-test	***Post-test***
Mean	0,411877395	0,545977011
Variance	0,036597259	0,043945469
t Stat	-8,445246317	
P(T<=t) one-tail	2,26665E-15	
t Critical one-tail	1,651971748	

The hypothesis H2 was confirmed on the significance level of 0,05.

Table 2. t-Test: Paired Two Sample for Means.

	Pre-test	***Post-test***
Mean	0,318783069	0,497354497
Variance	0,060220781	0,085802176
t Stat	-7,379424771	
P(T<=t) one-tail	1,71214E-12	
t Critical one-tail	1,651971748	

The hypothesis H3 was confirmed on the significance level of 0,05. However, difference according to gender in the third question was statistically significant ($p < 0.05$) in favour of the boys.

Table 3. F-Test Two-Sample for Variances; t-Test: Two-Sample Assuming Equal Variances

	Female	Male
Variance	0,06098555	0,058084
Observations	121	95
F	1,04994679	
P(F<=f) one-tail	0,40459556	
F Critical one-tail	1,38474046	

	Male	Female
Mean	0,35037594	0,293979
t Stat	0,55126713	
P(T<=t) one-tail	0,29101284	
t Critical one-tail	1,65200516	

The student solutions of the first question in the tests pointed to the formal acquisition of knowledge about linear dependence and inverse proportionality. Creating the correct table did not lead many students to the correct notion of a graphical representation of the explored dependence. 25 % of students in the first test and 24 % of students in the second test chose the wrong answer A. But 67 % of students chose the correct answer B in the second test, while only 44 % of students chose the correct answer in the first test. The analysis of students' solutions shows that students confuse the inverse proportionality with the property: "if one quantity increases, the other decreases." Each decreasing function has this property, and in our tests, two graphs of decreasing functions were among the possible answers in the questions 1. The argument that the discovered function is decreasing was also used the most by students in justifying their answers. The students used the analytical expression of the relationship between variables in the form $b = 12/a$ to a very small extent in justifying their answers.

In the question 2, in addition to the correct answer, the students most often chose the incorrect answer b (22 % in the pre-test, 11 % in the post-test). However, many students did not know how to approach the problem and did not have any solution (36 % in the pre-test, 17 % in the post-test). Analysis of student solutions has shown that many students are unable to work with a parameter. The letter in this question represented an unspecified negative number. Even students who decided to choose an answer often had difficulty understanding the meaning of the letters in the function rule. Student scores improved in the second test, but there was still a significant percentage of students who were unable to solve this question properly.

The third question was a word problem analogous to the above-mentioned task from PISA testing. The students should also write down the solution procedure for solving the question. The students preferred an arithmetic approach to solving this question. The students often performed arithmetic operations on given numbers without understanding the question. The example (see Fig. 2) shows one of the incorrect arithmetic solutions of the problem (the student's answer: At least 16.).

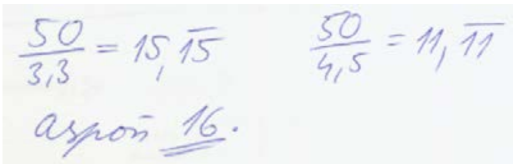

Figure 2. Incorrect arithmetic solution of the third question.

The success rate of students in solving this question in the pre-test was 32.87 %, in the post-test 50 %. The figure 3 shows one correct student's algebraic solution.

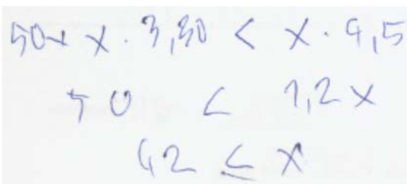

Figure 3. Correct algebraic solution of the third question.

The analysis of two test results showed that it is necessary to pay increased attention to the understanding of variables in the transition from arithmetic to algebra already in primary school.

5 INQUIRY LEARNING ACTIVITIES FOR DEVELOPMENT OF STUDENTS' INQUIRY SKILLS

The first example of an inquiry activity is selected from teaching materials focused on teaching the linear dependence. In the first stage of teaching, it is appropriate to work with simple models to demonstrate linear dependence. A suitable example for the simple assignment of corresponding values is a ruler which has on its two sides a scale for measuring lengths in centimetres and in inches. This simple tool inspired us to design and build a model for converting temperatures from the Celsius to Fahrenheit temperature scale. A model was created using GeoGebra. Interactive learning environment provides means for work with various representations: numeric data, dynamic diagram, table and a function rule. The dynamic diagram enables students to find a temperature expressed in °F corresponding to the temperature expressed in °C using sliders (see Fig. 4). Students are asked to enter obtained whole numbers into the second row of the table. Specifying of the temperature in the last column requests to discover a linear dependence between temperatures expressed in two scales and calculate the temperature expressed in °F corresponding to the temperature 50°C.

Temperature conversion between different temperature scales

Use sliders and a logical consideration to find corresponding temperatures in °F to given temperatures in °C.

°C	10	15	20	50
°F	50	59	68	122

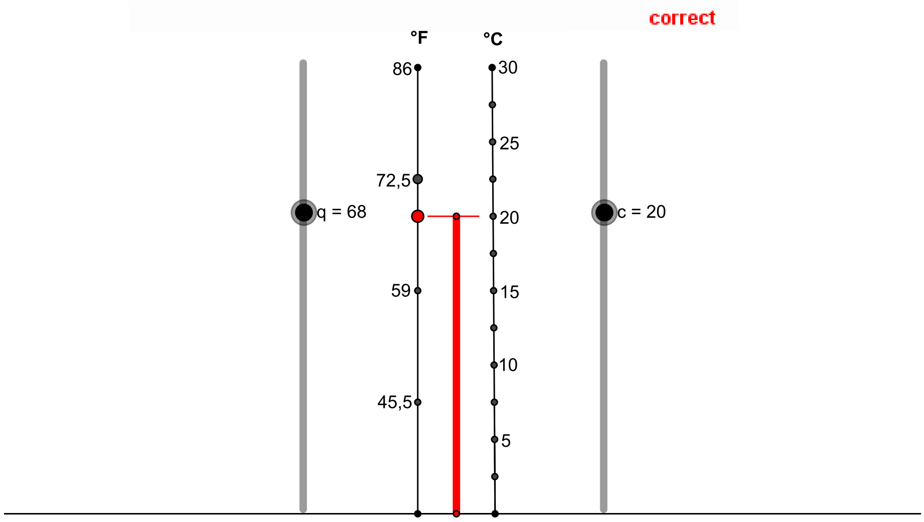

Figure 4. Dynamical model created in GeoGebra.

Interactive learning environment provides also minimal feedback for written results. If the student is not able to identify a type of a dependence between quantities, he/ she can use the auxiliary question attached to a check box: If the temperatures in °C change by the same values how change the temperatures expressed in °F? Identification of the correct type of functional dependence between explored quantities causes displaying a new task which requires creation of the rule of the investigated linear function. Figure 5 shows a properly completed formula for converting temperatures from the Celsius to the Fahrenheit temperature scale.

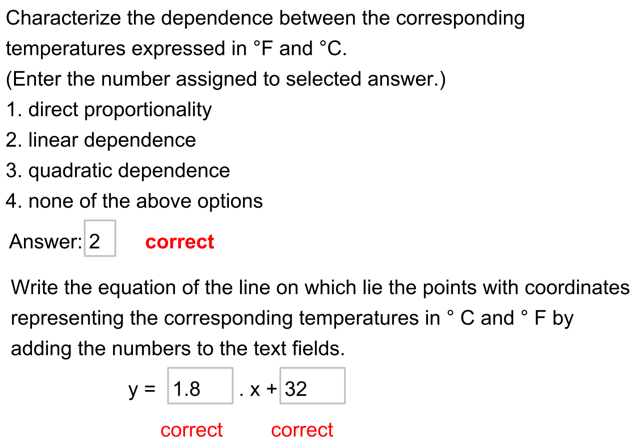

Figure 5. Implementation of feedback in the teaching material.

Teachers positively evaluated the possibilities of using the described learning activity in mathematics teaching. Students were particularly interested in the dynamic diagram, which they could use to search for pairs of matching temperatures in different temperature scales.

The analysis of students' solutions of the first question in the tests pointed out the students' shortcomings in identifying the inverse proportionality and in distinguishing the dependencies expressed by the graph of linear decreasing function and the graph of inverse proportionality. In our opinion, this question is a suitable idea for inquiry activity in teaching inverse proportionality. The teacher could give a motivational question at the beginning of the activity: *The rectangle ABCD with an area of 12 cm^2 is sketched in the coordinate system. When moving the vertex B along the x-axis, the rectangle ABCD is redrawn so that it always has an area of 12 cm^2. When redrawing rectangles, the vertex C leaves a trace on the drawing. Select the image that shows the path of the vertex C*.

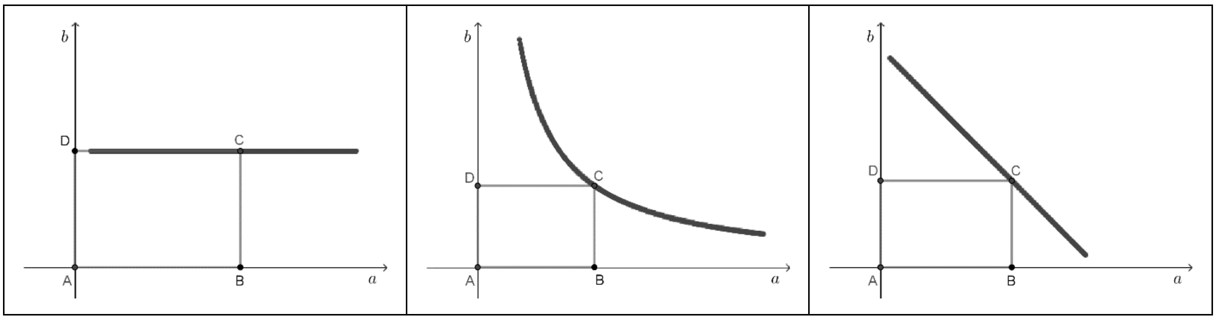

Figure 6. Options for a trace of the vertex C.

After choosing the answer, students should sketch several different rectangles with the area of 12 cm^2 in a square grid and write the lengths of the sides in a table. The teacher could also require the students to complete the data in ordered pairs: a = 1 cm, b = ?; a = ?, b = 3.5 cm; a = ?, b = 14 cm. In the next step based on the table and calculations, students should also symbolically write the dependence between the lengths of the rectangles *ABCD* with the area of 12 cm^2. At the end of the first part, the teacher would ask students to choose the correct graph for explored dependence from the four options listed in question 1 (see Fig. 7).

Students could then work with a dynamic construction. The vertex *B* can be shifted along the *x*-axis in the coordinate system, and the *y*-coordinate of the vertex *C* is automatically calculated according to the *x*-coordinate of the vertex *B*. Track recording is set for the rectangle *ABCD*.

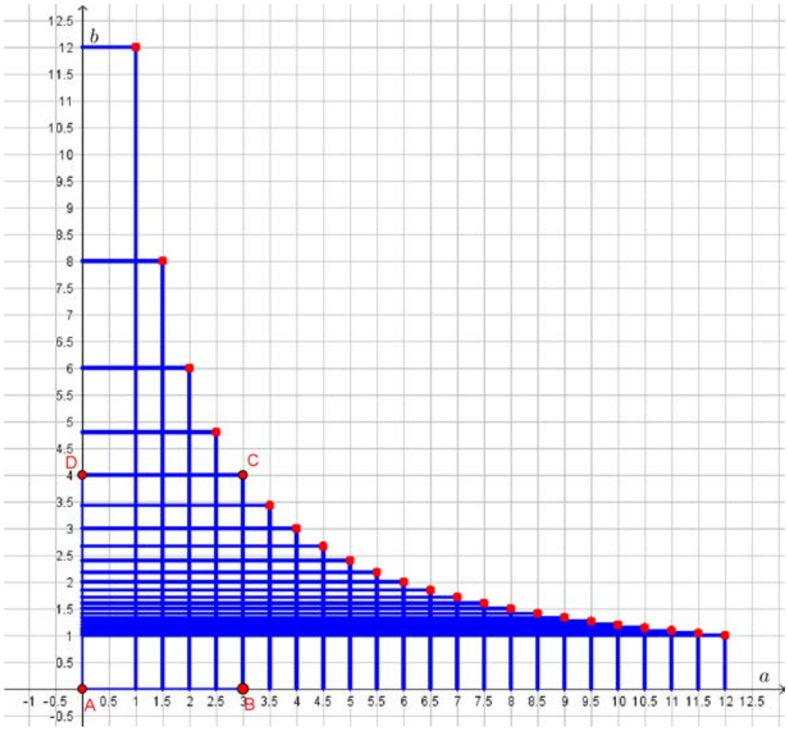

Figure 7. A construction of different rectangles ABCD with the area of 12 cm^2.

Based on the displayed rectangles, students should also verbally describe the investigated dependence: how many times the length of one side of the rectangle with the area of 12 cm^2 increases, so many times the length of the other side of this rectangle decreases. The size of the circumference of the rectangle *ABCD* can also be added to the given dynamic construction for each vertex *B*. Moving the vertex *B* would then also show the dependence of the circumference of the rectangles with the area of 12 cm^2 on the length of side *a*. Students could find rectangles (square) having the minimum circumference for the given area 12 cm^2. In a similar way, a teacher could create a dynamic construction to investigate the relationship between the side lengths of rectangles with the same circumference. Students can see for themselves that in this case the graphical representation of this dependence would correspond to a decreasing linear function (see the third possibility on figure 6).

6 CONCLUSIONS

Some beginning and less experienced teachers are often concerned about the application of IBL in mathematics teaching. The development of appropriate methodological and teaching materials is therefore the first important step for the successful implementation of inquiry approaches into mathematics and science education. The next important step is the organization of further teacher education, in which teachers are not only acquainted with the strategies for using the prepared teaching materials, but teachers are also led to plan and implement their own inquiry activities. The basics of working with the GeoGebra and the possibilities of its use in mathematics teaching should be included in further teacher education.

In the article, we presented two learning activities for the application of inquiry approaches to the mathematics teaching, which teachers can further develop according to the conditions in their classes, to be suitable for the development of inquiry students' skills. IBL should enable students to acquire new concepts and relationships by conducting inquiry work that lead students to explore the properties of objects and their mutual relationships. Experimentation and modelling allow students to gain new experience and knowledge and to think over using appropriate arguments to justify the discovered findings. IBL could help students in developing conceptual understandings of mathematical ideas. The application of IBL in the teaching mathematics and science in secondary schools could also help to increase students' interest in mathematics and science education and in the study of mathematics and science at universities.

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