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**NOVÉ TRENDY V STROJÁRSTVE  
NA PRAHU TRETIEHO TISÍCROČIA**

**1. sekcia  
MATEMATIKA A JEJ APLIKÁCIE  
V TECHNICKÝCH VEDÁCH**

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## On simple continuity

Jozef Doboš

Let  $X, Y$  be two topological spaces. Let  $\mathcal{F}$  be a class of functions  $f : X \rightarrow Y$ . A subset  $A$  of  $X$  with the property that whenever  $f \in \mathcal{F}$  is constant on  $A$ , then  $f$  must be constant on  $X$ , is said to be a *stationary set* for  $\mathcal{F}$ . Denote by  $\mathfrak{S}(\mathcal{F})$  the set of all stationary sets for the class  $\mathcal{F}$ . Observe that if  $\mathcal{F}_1 \subset \mathcal{F}_2$ , then  $\mathfrak{S}(\mathcal{F}_2) \subset \mathfrak{S}(\mathcal{F}_1)$ . (See [2], p.199.)

In this paper we give a complete characterization of stationary sets for the class of simply continuous functions.

A subset  $A$  of a topological space  $X$  will be said to be *simply open* if  $A = D \cup E$  where  $D$  is open and  $E$  is nowhere dense in  $X$ . Note that a set  $A$  is simply open if and only if  $\text{Fr } A$  (where  $\text{Fr } A = \text{Cl } A - \text{Int } A$ ) is nowhere dense. Let  $X, Y$  be two topological spaces. A function  $f : X \rightarrow Y$  is called simply continuous if for every open  $U$  in  $Y$ , the set  $f^{-1}(U)$  is simply open in  $X$ . In the sequel  $SC(X, Y)$  denotes the set of all simply continuous functions  $f : X \rightarrow Y$ . (See [1].)

### STATIONARY SETS FOR THE CLASS OF SIMPLY CONTINUOUS FUNCTIONS

We suppose that  $Y$  is a  $T_1$ -space which has at least two elements for the whole this paragraph.

**Theorem 1.** *Let  $X$  be a nonempty topological space. Let  $A$  be a subset of  $X$ . Then  $A \in \mathfrak{S}(SC(X, Y))$  if and only if the set  $X - A$  has not a nonempty simply open subset.*

*Proof.* Necessity. Deny. Suppose that there exists a nonempty simply open set  $B \subset X$  such that  $B \subset X - A$ . Choose  $u, v \in Y$  such that  $u \neq v$ . Define the function  $f : X \rightarrow Y$  as follows

$$f(x) = \begin{cases} u & \text{for } x \in B, \\ v & \text{otherwise.} \end{cases}$$

Then  $f \in SC(X, Y)$  and  $f$  is constant on  $A$ . But  $f$  is not constant on  $X$ .

Sufficiency. By contradiction. Suppose that  $X - A$  has not a nonempty simply open subset. Let  $f \in SC(X, Y)$ ,  $f(x) = c$  for  $x \in A$ ,  $f(b) \neq c$  for some  $b \in X - A$ . Let  $U$  be an open neighbourhood of the point  $f(b)$  such that  $c \notin U$ . Since  $f^{-1}(U)$  is nonempty simply open in  $X$ , by the assumption we have  $A \cap f^{-1}(U) \neq \emptyset$ . Thus  $c \in U$ , a contradiction.

**Corollary 1.** Let  $X$  be a nonempty topological space. Then every stationary set for the class  $SC(X, Y)$  is dense in  $X$ .

**Corollary 2.** Let  $X$  be a nonempty topological space. Then  $\mathfrak{S}(SC(X, Y)) = \{X\}$  if and only if every singleton (in  $X$ ) is simply open.

**Definition.** A topological space  $X$  is a  $T_{\frac{1}{2}}$ -space if and only if  $\text{Cl}(\{a\}) - \{a\}$  is closed for each  $a \in X$  (see [5]).

**Theorem 2.** Let  $X$  be a nonempty  $T_{\frac{1}{2}}$ -space. Then  $\mathfrak{S}(SC(X, Y)) = \{X\}$ .

*Proof.* Let  $a \in X$ . Suppose that  $\{a\}$  is not open. We shall prove that  $\{a\}$  is nowhere dense in  $X$ . Since  $X$  is a  $T_{\frac{1}{2}}$ -space, the set  $W = \{a\} \cap \text{Int Cl}\{a\}$  is open. Then  $W = \text{Int } W = \text{Int}\{a\} = \emptyset$ . Hence  $a \notin \text{Int Cl}\{a\}$ , therefore  $\text{Int Cl}\{a\} = \emptyset$ . By Corollary 2 the desired property is ensured.

The following example shows that the assumption " $T_{\frac{1}{2}}$ -space" in Theorem 2 cannot be replaced by the assumption " $T_0$ -space".

**Example.** Let  $X$  be an infinite set,  $a \in X$ . Define the open sets of a topology  $\tau$  on  $X$  to be  $\emptyset$  and all sets of the form  $A \cup \{a\}$  where  $A \subset X$ ,  $X - A$  is finite. Then  $(X, \tau)$  is a  $T_0$ -space, but  $X - \{a\} \in \mathfrak{S}(SC(X, Y))$ .

#### STATIONARY SETS FOR SOME OTHER GENERALIZATIONS OF CONTINUITY

A subset  $A$  of a topological space  $X$  will be said to be *pseudo-open* if  $A = G \cup H$  where  $G$  is open and  $H$  is a set of the first Baire category.

Let  $X, Y$  be two topological spaces. A function  $f : X \rightarrow Y$  is called *pseudo-continuous* if for every open  $U$  in  $Y$ , the set  $f^{-1}(U)$  is pseudo-open in  $X$ . (See [4].)

Let  $X$  be a topological and  $Y$  a metric spaces (with the metric  $d$ ). A function  $f : X \rightarrow Y$  is said to be *cliquish* at the point  $p \in X$  if for each neighbourhood  $U(p)$  of the point  $p$  and each  $\varepsilon > 0$  there exists a nonempty open set  $U \subset U(p)$  such that  $d(f(x), f(y)) < \varepsilon$  holds for each two points  $x, y \in U$ . A function  $f : X \rightarrow Y$  is said to be *cliquish on  $X$*  if it is cliquish at each point  $a \in X$ . (See [6].)

Let  $X, Y$  be two topological spaces, let  $f : X \rightarrow Y$ . Denote by  $C_f$  ( $D_f$ ) the set of all continuity (discontinuity) points of the function  $f$ . A function  $f : X \rightarrow Y$  is called *pointwise discontinuous* if  $C_f$  is dense in  $X$ .

In the sequel  $PC(X, Y)$ ,  $Q^*(X, Y)$ , and  $PD(X, Y)$  denote the sets of all functions  $f : X \rightarrow Y$  which are pseudo-continuous, cliquish on  $X$ , and pointwise discontinuous, respectively.

**Theorem 3.** Let  $X$  be a nonempty topological space and  $Y$  a metric space which has at least two elements. Then

$$\mathfrak{S}(PC(X, Y)) = \mathfrak{S}(Q^*(X, Y)) = \mathfrak{S}(PD(X, Y)) = \mathfrak{S}(SC(X, Y)).$$

*Proof.* Since  $PD(X, Y) \subset Q^*(X, Y) \subset PC(X, Y)$  (see [3] and [4]), we have

$$\mathfrak{S}(PC(X, Y)) \subset \mathfrak{S}(Q^*(X, Y)) \subset \mathfrak{S}(PD(X, Y)).$$

Now, we shall prove that  $\mathfrak{S}(PD(X, Y)) \subset \mathfrak{S}(SC(X, Y))$ . Let  $A \subset X$ ,  $A \notin \mathfrak{S}(SC(X, Y))$ . Then by Theorem 1, there exists a nonempty simply open set  $S \subset X$  such that  $S \subset X - A$ . Choose  $u, v \in Y$  such that  $u \neq v$ . Define the function  $f : X \rightarrow Y$  as follows

$$f(x) = \begin{cases} u & \text{for } x \in S, \\ v & \text{otherwise.} \end{cases}$$

Since  $D_f \subset \text{Fr } S$ , the set  $D_f$  is nowhere dense in  $X$ . Then  $f \in PD(X, Y)$ ,  $f$  is constant on  $A$ , but  $f$  is not constant on  $X$ . Thus  $A \notin \mathfrak{S}(PD(X, Y))$ .

Finally, we shall prove that  $\mathfrak{S}(SC(X, Y)) \subset \mathfrak{S}(PC(X, Y))$ . Let  $A \subset X$ ,  $A \notin \mathfrak{S}(PC(X, Y))$ . Then there exists  $g \in PC(X, Y)$  such that  $g(x) = c$  for some  $x \in A$ ,  $g(b) \neq c$  for some  $b \in X - A$ . Thus the set  $X - g^{-1}(c) = g^{-1}(Y - \{c\})$  is nonempty pseudo-open. Choose a nonempty simply open set  $B \subset X$  such that  $B \subset X - g^{-1}(c)$ . Choose  $a \in Y$  such that  $a \neq c$ . Define the function  $h : X \rightarrow Y$  as follows

$$h(x) = \begin{cases} a & \text{for } x \in B, \\ c & \text{otherwise.} \end{cases}$$

Then  $h \in \mathfrak{S}(SC(X, Y))$ ,  $h$  is constant on  $A$ , but  $h$  is not constant on  $X$ . Therefore  $A \notin \mathfrak{S}(SC(X, Y))$ .

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